AP CALCULUS SUMMER REVIEW WORK

The following problems are all **ALGEBRA** concepts you must know cold in order to be able to handle Calculus. Most of them are from Algebra 2, some are from Pre-Calc. This packet is to make sure you are prepared for next year. Some of these ideas you may have forgotten over the past two years…now is the time to start remembering. If you have no clue what to do you can look things up online, talk to friends, or even ask me. This packet should be done before the first day of school. I will NOT be collecting it but you will **have a TEST on the first day of school**. These problems should be easy and it is your responsibility to make sure they are. I am here to help you. The sooner you get comfortable asking questions the better. A calculator should NOT be used to do these problems

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Factoring

One of the most important mathematical procedures you will do all year in calculus is factoring, most of the time to find the zeros of a function. There are two basic ways you will need to know how to factor:

Pulling out a common factor…

$3x^3 - 6x$

Factors to

$3x(x^2 - 2)$

And what I like to call "unfoiling"…

$x^2 + x - 6$

Factors to

$(x-2)(x+3)$

You will also see a combination of the two…

$2x^2 + 4x - 6$

$2(x^2 + 2x - 3)$

$2(x+3)(x-1)$

Here are some example problems to practice. Factor each.

1. $x^2 + 3x + 2$

2. $x^2 - 5x - 14$

3. $x^2 - 6x + 8$

4. $x^2 - 6x + 5$

5. $x^4 - x^3$

6. $6x^2 - 2x^3$

7. $-6x^2 - 3x$

8. $x^4 - 4x^2$

9. $2x^4 + 16x$

10. $-x^3 - 9x$

11. $2x^2 + 10x + 12$

12. $3x^2 - 18x + 15$

Zeros

Most of the time we will factor in order to find the zeros of a function (when a function is equal to zero).

A few things to remember: if you take the square root of a number the answer is plus or minus
you cannot take the square root of a negative number
answers can be left as roots (i.e. $\sqrt{5}$)

Find the zeros of each of these equations.

1. $2x-4=0$ 2. $2x^2-18=0$ 3. $5-x^3=0$

1. $x^2-4=0$ 2. $x^2+4=0$ 3. $x^2-2x-15=0$

Equations of lines

Now let's take a trip back to eighth grade, where all this algebra began.

Slope intercept form is what you guys are most use to… $y=mx+b$.

We will be using point-slope form next year in Calculus… $y-y_1=m(x-x_1)$.

You should also know how to find slope… $\frac{y_2-y_1}{x_2-x_1}$ **NOTE: Slope is the RATE OF CHANGE of a line. Anytime you are asked to find a RATE you are just being asked to find a SLOPE!**

Parallel lines have the same slope.

Last, perpendicular lines have opposite reciprocal slopes… $\frac{2}{3}$ *and* $-\frac{3}{2}$ are perpendicular slopes.

In order to write the equation of a line in point-slope form all you need is a point (either point if there are two of them) and the slope. Write the equation of each line described below in point-slope form.

1. Through the points (1, 2) and (3, 6).
2. Through the points (-1, 6) and (5, -2).
3. Through the point (1, 2) and parallel to $y=\frac{1}{2}x+3$.
4. Through the point (-3, -4) and parallel to $y=3x+1$.
5. Through the point (2, 6) and parallel to $3y+2x=2$.
6. Through the point (1, 2) and perpendicular to $y=\frac{2}{3}x-4$.
7. Through the point (3, 5) and perpendicular to $3y+2x=2$.

Negative Exponents & Fraction Exponents

You will see negative exponents and exponents that are fractions throughout the year.

Negative Exponents tell us to move the expression to the top or bottom of a fraction.

Example: $x^{-2} \rightarrow \frac{1}{x^2}$ or $\frac{2}{3x^{-3}} \rightarrow \frac{2x^3}{3}$ Notice in this example the 3 did not move up with the x. The 3 was not raised to the negative exponent, only the x was.

Evaluate: $2^{-3} \rightarrow \frac{1}{2^3} \rightarrow \frac{1}{8}$

Exponents that are Fractions – The top is the power and the bottom is the root. You may recall from Algebra 2 that $\sqrt{x}$ can be written as $x^{1/2}$. Here are some other examples of how to rewrite expressions with exponents that are fractions.

Example: $x^{1/3} \rightarrow \sqrt[3]{x}$ $\quad x^{2/3} \rightarrow (\sqrt[3]{x})^2$ $\quad x^{3/4} \rightarrow (\sqrt[4]{x})^3$ $\quad x^{3/2} \rightarrow (\sqrt{x})^3$

Evaluate: $27^{2/3} \rightarrow (\sqrt[3]{27})^2 \rightarrow 3^2 \rightarrow 9$ $\quad 4^{3/2} \rightarrow (\sqrt{4})^3 \rightarrow 2^3 \rightarrow 8$

Combining The Two

Example: $x^{-1/2} \rightarrow \frac{1}{x^{1/2}} \rightarrow \frac{1}{\sqrt{x}}$ $\quad 4^{-1/2} \rightarrow \frac{1}{4^{1/2}} \rightarrow \frac{1}{\sqrt{4}} \rightarrow \frac{1}{2}$

Simplify by getting rid of all negative exponents.

1. $2x^{-3}$
2. $\frac{3x^{-4}}{2}$
3. $\frac{5}{2x^{-1}}$
4. $-\frac{3x^{-2}}{5}$

Evaluate each.

1. $4^{1/2}$
2. $8^{2/3}$
3. 2^{-3}
4. $16^{-1/2}$
5. $16^{3/2}$
6. $4^{-3/2}$

ln & e

In Pre-Calc you learned about **natural log** (ln) and **e**. These are two very important concepts that we will be using in calculus.

Natural Log can be used to bring down an exponent… $\ln 3^x \to x\ln 3$

We can use this idea to solve an equation, like… $5^x = 10$

Here are the steps I would use…

$\ln 5^x = \ln 10$ **take the ln of both sides**

$x\ln 5 = \ln 10$ **we must realize that ln(5) and ln(10) are numbers**

$x = \dfrac{\ln 10}{\ln 5}$ **This is our answer.**

e is a number (2.7182…). It is important to realize no matter what power you raise it to it will always be a positive number.

Ln and e are opposites. Together they cancel each other out.

$\ln e^5 = 5$ $\quad e^{\ln 6} = 6$

We can use this idea to solve equations involving either of these…

$e^{3x} - 5 = 0$ **add over 5**

$e^{3x} = 5$ **cancel e by taking ln of both sides**

$\ln(e^{3x}) = \ln(5)$

$3x = \ln(5)$ **divide by 3**

$x = \dfrac{\ln(5)}{3}$ **answer**

$\ln(2x) + 8 = 0$ **subtract over 8**

$\ln(2x) = -8$ **raise both sides with base of e**

$e^{\ln(2x)} = e^{-8}$

$2x = e^{-8}$ **e cancels out ln**

$x = \dfrac{e^{-8}}{2}$ **remember e^{-8} is a #**

You must get ln or e by itself before you can cancel them out (if there is a number in front divide it over first).

Find the zeros of each equation using the rules of ln & e.

1. $e^x - 4 = 0$

2. $5 - 10e^x = 0$

3. $e^{3x} - 10 = 0$

4. $6e^{-5x} - 12 = 0$

5. $e^{2x^2} - 18 = 0$

6. $3 - 3e^{x^2} = 0$

7. $\ln(x) + 5 = 0$

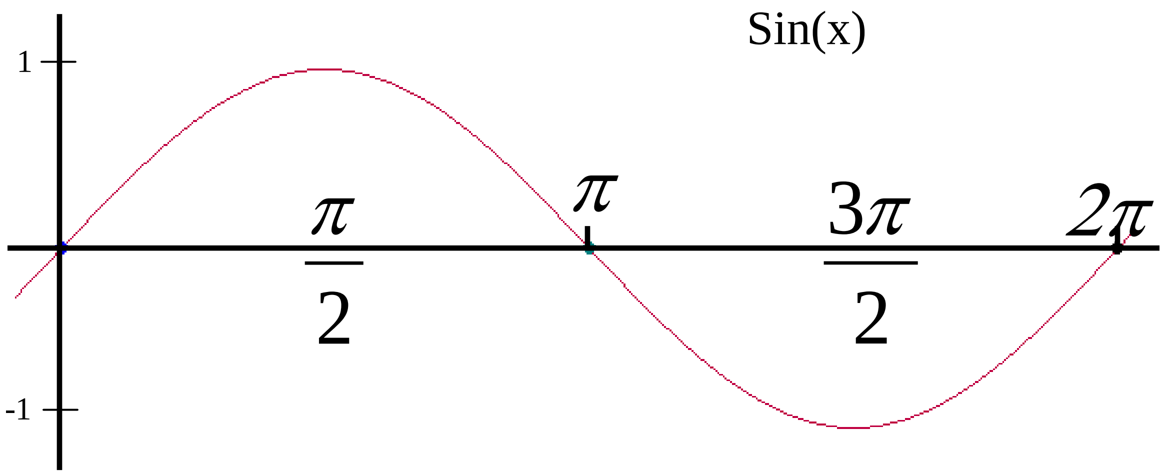
8. $3\ln(x) - 2 = 0$

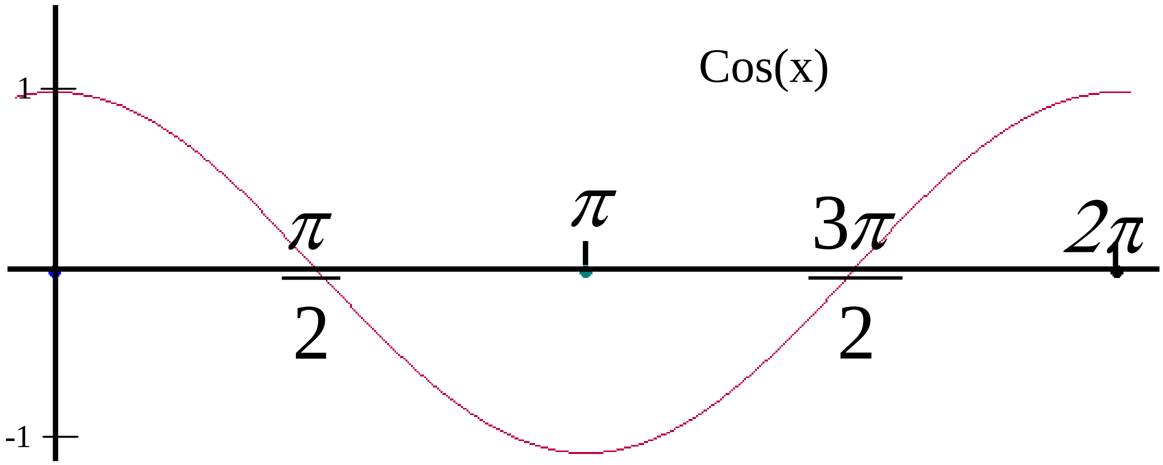
9. $\ln(x^2) + 6 = 0$

10. $10 - 5\ln(2x) = 0$

Trig

You ***MUST*** be able to graph two trig functions: sin(x) and cos(x) over 1 period.



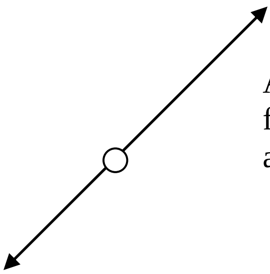


Using these graphs you will be able to answer simple trig questions without a calculator. Use the points on the graphs to find the answers to the following.

1. $\sin(0) =$
2. $\cos(0) =$
3. $\sin(\pi) =$
4. $\sin\left(\frac{\pi}{2}\right) =$
5. $\cos(\pi) =$
6. $\cos\left(\frac{3\pi}{2}\right) =$
7. $\sin\left(\frac{3\pi}{2}\right) =$
8. $\cos(2\pi) =$

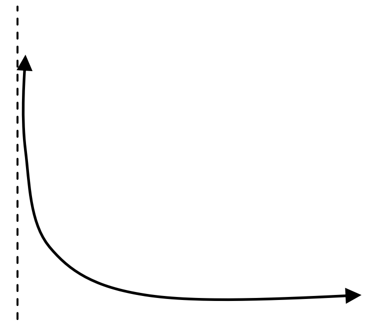
<u>Points of Discontinuity</u>

In algebra 2 you learned about points of discontinuity (points where a function is not continuous). The two specific examples you learned where holes and vertical asymptotes.



A hole is where a function is undefined at one point.

A vertical asymptote is where a function goes up or down to infinity, never reaching a certain x value



Both of these occur due to the impossibility of dividing by zero.

The zeros of the bottom of a fraction are points of discontinuity. If the zero can be canceled with the top then it is a hole. If the zero cannot be canceled out then it is a vertical asymptote.

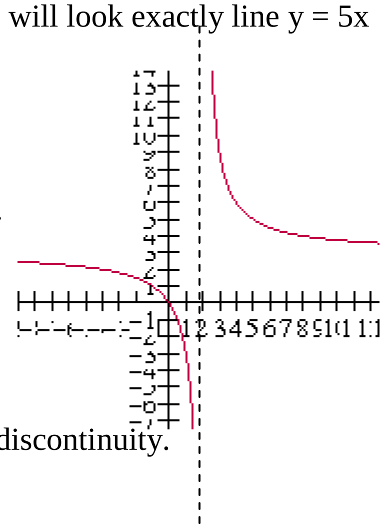
Examples:

$y = \frac{5x^2}{x}$ The zero of the bottom is 0. This tells me x=0 is a point of discontinuity.

Since it can be canceled… $y = 5x$ …the discontinuity is a hole. The function $y = \frac{5x^2}{x}$ will look exactly line y = 5x but with a hole at x = 0.

$y = \frac{3x}{x-2}$ The zero of the bottom is 2. This tells me x = 2 is a point of discontinuity.

Since it CANNOT be canceled with the top x = 2 is a vertical asymptote.



$y = \frac{(x-3)(x+4)}{(x+4)(x-1)}$ The zeros of the bottom are x = -4 and 1. Those are the points of discontinuity.

x + 4 can be canceled with the top therefore x = 4 is a hole.

x – 1 cannot be canceled therefore x = 1 is a vertical asymptote.

Determine the points of discontinuity of each function and if they are a hole or a VA.

1. $y = \frac{3}{x-5}$

2. $y = \frac{x-5}{x-3}$

3. $y = \frac{x+2}{x(x+2)}$

4. $y = \frac{x^3}{1-x}$

5. $y = \frac{5}{x^2-4}$

6. $y = \frac{(x-2)(x+3)}{(x+3)(x+1)}$

7. $y = \frac{x^2+5x+6}{x^2+6x+8}$ (hint: factor top and bot)